

# Precision Tracking Control of a Piezoelectric-Actuated System

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**Abstract--In this paper, precision tracking control of piezoelectric-actuated systems is discussed. In order to obtain precision tracking control, Prandtl-Ishlinskii (PI) model is used to model the hysteresis nonlinearity. Then, the inverse PI model is used to reduce the hysteresis nonlinearity and a sliding-mode controller is used to compensate the remaining nonlinear uncertainty and disturbances. The piezoelectric-actuated system is modeled as a linear model coupled with a hysteresis. The linear model is identified then it is used to design the sliding-mode controller. Finally, experimental results are presented to verify the usefulness of this method.**

## I. INTRODUCTION

Piezoelectric actuators are becoming increasingly important in today's positioning technology due to the requirements of nanometer resolution in displacement. It is well known that the piezoelectric actuator has many advantages [1] such as: 1) there are no moving parts; 2) the actuators can produce large forces; 3) they have almost unlimited resolution; 4) the efficiency is high; and 5) response is fast. However, it also has some bad characteristics such as: 1) hysteresis behavior; 2) drift in time; 3) temperature dependence. Hysteresis characteristics are generally nondifferentiable nonlinearities and usually unknown, this often limits system performance via, e.g., undesirable oscillations or instability. Therefore, it is difficult to obtain an accurate trajectory tracking control.

Recently, several methods have been reported for the trajectory tracking control of a piezoelectric-actuated system. Ge and Jouaneh [2,3] used a combination of a proportional integral derivative (PID) feedback controller with a feedforward controller that included the Preisach model of hysteresis. Their experimental results showed that the tracking performance was improved greatly. However, the result in [2] is only valid for a sinusoidal trajectory and the method in [3] needs to train the model by using reference input before the control started. Ku *et al.* [7] combined a PID feedback controller with an adaptive neural network feedforward controller to control a nanopositioner that was actuated by a piezoelectric actuator. Cruz- Hernandez and Hayward [1] proposed a variable phase method. They utilized an operator to shift the

periodic input signal by a phase angle that depended on the amplitude of the input signal, then used this operator to reduce the hysteresis nonlinearity. Huang and Lin [4] proposed a new hysteresis model based on two first-order transfer functions in parallel with two parameters determined from experiment. Adaptive control is also an approach to the inverse control of plants with hysteresis behavior. Tao and Kokotovic [9] developed an adaptive hysteresis inverse and cascaded it with the system so that the effects of hysteresis nonlinearity could be reduced. Xu [11] utilized an adaptive neural network inverse controller to compensate the hysteretic behavior of a piezoelectric actuator and a PI controller in the outer loop to overcome the remaining nonlinear uncertainty. Furthermore, Hwang *et al.* [6] utilized an offline learned neural network model to reduce the effect of hysteresis then designed a discrete-time variable structure controller to overcome the remaining uncertainty. They also reinforced this method [5] by using a recurrent neural network to improve the control performance. However, the computation burden of the controller that was designed by their method is heavy. In the literature [12], Shen *et al.* utilized integer sliding-mode controller to compensate the nonlinearity and disturbances in piezoelectric actuated systems.

In this paper, Prandtl-Ishlinskii (PI) [13,14] model is used to model the hysteresis nonlinearity. Then, the inverse PI model is used to reduce the hysteresis nonlinearity and a sliding-mode controller is used to compensate the remaining nonlinear uncertainty and disturbances. The main advantages of PI model over the Preisach model are that it is less complex and its inverse can be computed analytically. In this study, the sliding-mode uncertainty (disturbance) estimation and compensation scheme [8,10,12] is used to design the feedback controller. Finally, this design method was applied to the motion control of a nano-stage. The experimental results are presented that verify the usefulness of this method.

## II. EXPERIMENTAL SETUP

The experimental apparatus of this study is a one-axis nano-stage. Actuation of this nano-stage is done with a piezoelectric actuator (PSt150/7/20 Vs12, Piezomechanik

GmbH) with a nominal expansion of  $20 \mu\text{m}$  (at 150V). Capacitance-type gap sensors (D-015, Piezomechanik GmbH) are used for position measurement. The measured range of the gap sensor is extended by a factor of three. Therefore, total ranges of the gap sensors are  $45 \mu\text{m}$  with a sensitivity of  $0.222 \text{ V} / \mu\text{m}$ . Control of this nano-stage is done with a controller board with a Power PC central processing unit (DS1103, dSPACE GmbH). The sampling rate of the control algorithm was 10 KHz. The resolution of A/D converters is 16-bit.

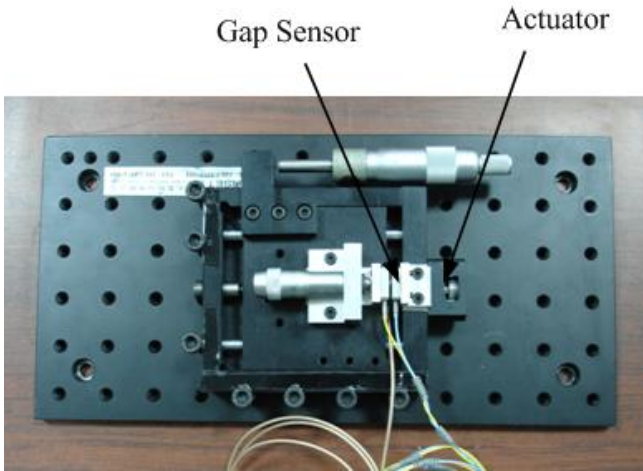


Fig. 1. Photograph of the nano-stage.

### III. SYSTEM MODELING AND CONTROLLER DESIGN

This section describes the way to model the piezoelectric actuated system and how to design the feedback controller. Firstly, PI hysteresis model is used to model the hysteresis nonlinearity of actuator and the inverse of this model is used to cancel out the hysteresis nonlinearity. Then the linearized model of the piezoelectric actuated system is estimated and an integral sliding mode controller is designed to compensate the remaining uncertainties and disturbances.

#### 3.1 Prandtl-Ishlinskii Hysteresis Model

The elementary operator in the PI hysteresis model is a backlash operator (see Fig. 2). A backlash operator is defined by

$$y(k+1) = H_r[x, y_0](k+1) = \max\{x(k+1) - r, \min\{x(k+1) + r, y(k)\}\} \quad (1)$$

where  $x$  is the control input,  $y$  is the output of actuator,  $r$  is the magnitude of backlash. The initial condition of (1) is given by

$$y(0) = \max\{x(0) - r, \min\{x(0) + r, y_0\}\}$$

Where initial state  $y_0 \in \mathbb{R}$ , and is usually but not necessarily initialized to 0.

Hysteretic nonlinearities can be modeled by a linearly weighted superposition of many backlash operators with different threshold and weight values,

$$H[x](k) = \sum_{i=0}^n w_i H_{r_i}[x, y_{0i}](k) \quad (2)$$

where  $w_i$  are the weightings,  $r_i$  are the threshold values with  $0 = r_0 < r_1 < \dots < r_n$  and  $y_{0i}$  are the initial states.

The control input threshold values  $r_i$  are usually chosen to be equal intervals.

To find the parameters of hysteresis model, the responses of the actuator must be measured experimentally. Then the values of  $r_i$  are selected and the weight parameters  $w_i$  are found by doing a least square fit of equation (2) to the response data of actuator. Fig. 3 shows the hysteresis response of the actuator and its PI model ( $n=14$ ).

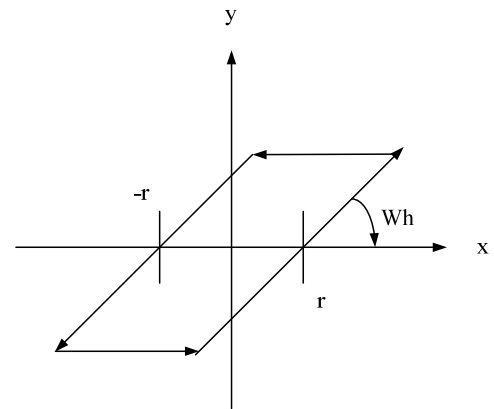


Fig. 2. Backlash operator

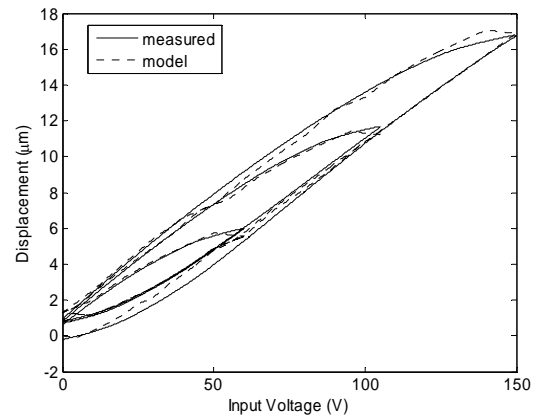


Fig. 3. Measured hysteresis nonlinearity and its model.

#### 3.2 Inverse Model of Hysteretic Nonlinearities

The inverse of the model (2) is also a PI type and is given by

$$H^{-1}[y](t) = \sum_{i=0}^n w'_i H_{r'_i}[y, y'_{0i}](t) \quad (3)$$

The inverse model parameters can be found by

$$w'_0 = \frac{1}{w_0}; \quad w'_i = -\frac{w_i}{(w_0 + \sum_{j=1}^i w_j)(w_0 + \sum_{j=1}^{i-1} w_j)},$$

$$y'_{0i} = \sum_{j=0}^i w_j y_{0i} + \sum_{j=i+1}^n w_j y_{0j},$$

$$r'_i = \sum_{j=0}^i w_j (r_i - r_j); \quad i = 0 \dots n$$

The inverse model (3) can be used to cancel out the hysteresis nonlinearity of piezoelectric actuator as shown in Fig. 4. Fig. 5 shows the inverse compensated result. It can be seen that most hysteresis nonlinearity is compensated.

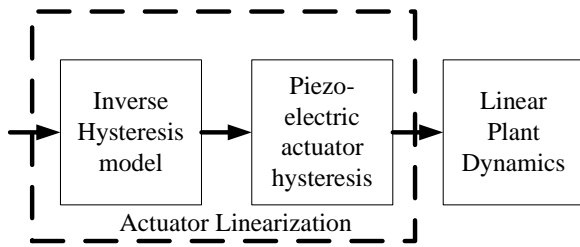


Fig. 4. Piezoelectric actuator linearization.

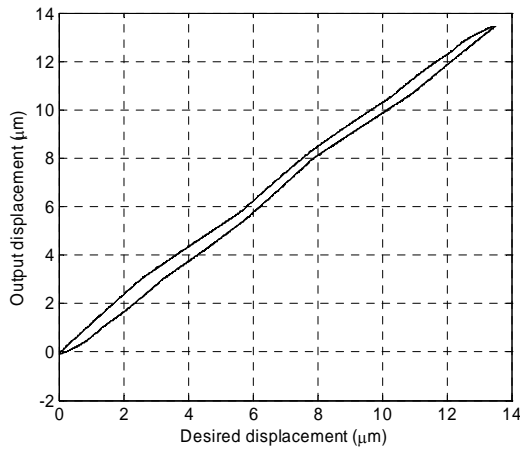


Fig. 5. Result of inverse compensation.

### 3.3 Open Loop Characteristics and Model

In order to design the feedback controller properly, the open-loop characteristics of the piezoelectric actuated systems are investigated. Firstly, the frequency-response experiments are conducted. When measuring the frequency response, a bias voltage was added to push the stage platform to the center of the moving range. Then, a random excitation signal was sent to the piezoelectric actuator and the displacement is measured by the gap sensor.

The test results are depicted in Fig. 6. As seen in Fig.6, the bandwidth is about 86 Hz. A linear dynamic model, represented as a third order transfer function was

curve-fitted to the measured frequency response. The response of the model also shows on Fig. 6. The poles of the model are -438 and  $-5913 \pm j31415$  respectively. The dominant pole is -438 and the others poles are more than 10 times faster than this pole. Therefore, it is possible to model this system by a first order transfer function.

Base on the results of investigation, the linear plant dynamics of this piezoelectric-actuated system can be modeled by a first order uncertain linear system. Thus, the model of this system can be modeled as the block diagram shows in Fig. 7. Where  $u$  is the input of the inverse model,  $d$  represents the disturbance and the first order differential equation

$$T(1 + \Delta(t))\dot{y} + y = v, \quad (4)$$

describes the dynamic behavior of the system. Where  $T$  is the nominal time constant and  $\Delta(t)$  represents the uncertainty. Parameter  $T$  and the bound of  $\Delta(t)$  can be determined by doing step response tests at various working points.

From Fig. 7,  $v$  can be represented as

$$v = u + N(t) + d(t), \quad (5)$$

where  $N(t)$  represents the remaining nonlinear uncertain part of the hysteresis. From (4) and (5), the following dynamic equation can be obtained:

$$\dot{y} = -\frac{y}{T} + \frac{u}{T} + \phi(t) \quad (6)$$

where

$$\phi(t) = \frac{\Delta(y-u) + N + d}{T(1 + \Delta)}$$

represents the disturbance and uncertainties.

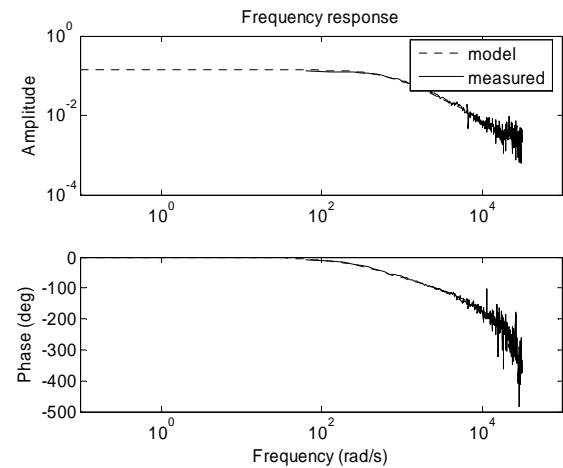


Fig. 6. Frequency response of the system

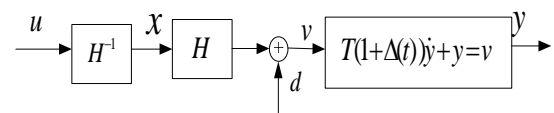


Fig. 7. Model of the piezoelectric-actuated system.

### 3.4 Integral Sliding-Mode Controller Design

This subsection describes how to design the integral sliding-mode controller. In this study, the sliding mode disturbance (uncertainty) estimation and compensation scheme is applied to design the closed-loop controller for the piezoelectric-actuated system.

Let  $y_d$  be the desired displacement, which may be time varying. Define

$$e = y_d - y \quad (7)$$

as the tracking error. From (6) and (7), the error dynamics can be obtained as

$$\dot{e} = \dot{y}_d - \dot{y} = \dot{y}_d + \frac{y}{T} - \frac{u}{T} - \phi(t). \quad (8)$$

Let the control law be

$$u = y + T(\lambda e + \dot{y}_d) + u_d \quad (9)$$

where  $\lambda$  is the feedback gain to be designed so that the error dynamic will have the desired response while the system is free of disturbance and uncertainty, and  $u_d$  is the uncertainty and the disturbance compensation component yet to be determined by the sliding mode estimator.

Defining the switching function as

$$S = z - e \quad (10)$$

with

$$\dot{z} = -\lambda e - \frac{1}{T}u_d + \psi, \quad z(0) = e(0) \quad (11)$$

where  $z$  is the state variable of this auxiliary process,  $\psi$  is the switching action assigned as

$$\psi = -\eta \text{sign}(S), \quad \text{sign}(S) = \begin{cases} 1 & \text{if } S > 0 \\ -1 & \text{if } S < 0 \\ 0 & \text{if } S = 0 \end{cases} \quad (12)$$

and the positive constant  $\eta$  satisfies

$$\eta > |\phi(t)| \quad (13)$$

Ensuring a sliding regime  $S=0$  requires consideration of the Lyapunov candidate  $v = 0.5S^2$ . Differentiating  $V$  with respect to time and substituting (8-11) to obtain

$$\dot{V} = S(\dot{z} - \dot{e}) = S[-\eta \text{sign}(S) + \phi(t)] \quad (14)$$

From (13) and (14), it is seen that

$$\dot{V} < 0 \quad \text{if } S \neq 0 \quad (15)$$

Thus the sliding condition is satisfied. Note that  $z(0) = e(0)$ , therefore

$$S = 0 \quad \text{for } t = 0 \quad (16)$$

From (15) and (16), it can be concluded that the sliding mode exists at all times, *i.e.*,

$$\dot{S} = 0 \quad \text{for all } t \geq 0 \quad (17)$$

Denote the equivalent value of  $\psi$  as  $\psi_{eq}$ . Since  $\dot{S} = 0$ ,  $\psi_{eq}$  can be determined from (8), (10) and (11):

$$\psi_{eq} = -\phi \quad (18)$$

This means that the equivalent value of  $\psi$  equals the uncertainties and disturbances. By selecting  $u_d = T\psi_{eq}$ , the uncertainties and disturbances can be compensated. It was shown in [10] that the equivalent  $\psi_{eq}$  is equal to the average value measured by a first-order linear filter with the switched action as its input. Therefore,  $u_d$  can be written as

$$u_d = T\psi_{av} = T\psi_{eq} \quad (19)$$

with

$$\tau\dot{\psi}_{av} + \psi_{av} = \psi. \quad (20)$$

The time constant  $\tau$  should be made small enough that the plant and disturbance dynamics are allowed to pass through the filter without significant phase delay. Substituting (19) and (9) into (8) yields

$$\dot{e} + \lambda e = -\psi_{eq} - \phi, \quad (21)$$

which is equivalent to  $\dot{e} + \lambda e = 0$ . This equation represents the desired error dynamics.

## IV. EXPERIMENTAL RESULTS

In order to identify the nominal time constant  $T$  and estimate the bound of  $\Delta$ , step response tests at various working points are executed. From the test results, it was found that the time constant of this stage lies between 2.2 and 3.5 ms. When designing the controller,  $\lambda$  is chosen as large as possible to obtain wider bandwidth. The final controller parameters are chosen as follows:  $\lambda = 550$ ,  $\eta = 20$  and  $\tau = 0.0018$ .

In order to check the performance of this control method, sinusoidal waves with  $9\mu m$  amplitude and different frequencies range from 1-20Hz were used to test the tracking performance. Fig. 8 depicts the tracking results of 10Hz signal. Table 1 summarizes the maximum tracking errors of tracking test at different frequencies. Comparison was made with the integral sliding-mode controller (without inverse model compensation). It can be found from Table 1 that the inverse model compensation can improve the tracking accuracy significantly. Fig. 9 shows the tracking results of a multi-frequency, nonstationary dynamic motion profile (modulated 2 and 10 Hz sinusoids with varying amplitudes). The maximum tracking error is  $0.039\mu m$ . It can be seen that the output still can track the reference very well. For determining the precision of the tracking control, a 5Hz sine wave with 4nm amplitude was used to command the stage. Fig. 10 shows the trajectory following results of this stage to 4nm amplitude input. For very small amplitude of reference position of 4nm which is almost the same as the magnitude of noise, the measured position still tracked the reference fairly well. It can be concluded that the tracking resolution is about 4nm. In order to determine the closed-loop bandwidth of this stage, a  $9\mu m$  magnitude sinusoidal signal was command to this nano-stage. The frequency of the command motion was slowly increased until an attenuation of 0.707 in the

tracking magnitude. The measured bandwidth was approximately 80 Hz.

### V. CONCLUSIONS

This article reported the precision tracking control of a piezoelectric actuated motion stage. Firstly, Prandtl-Ishlinskii (PI) model is used to model the hysteresis nonlinearity of the actuator. Then, the inverse PI model was used to reduce the hysteresis nonlinearity and a sliding-mode controller was used to compensate the remaining nonlinear uncertainty and disturbances. The most important features of PI model are that it is less complex and its inverse can be computed analytically. Experiments on tracking the sinusoidal waveforms and nonstationary dynamic motion profile were carried out. From the result it can be seen that the performance of this controller is good and 4nm tracking resolution can be obtained.

### ACKNOWLEDGEMENTS

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Table 1. Results of tracking test.

Frequency (Hz)	Maximum Tracking Error ( $\mu m$ )	
	Without Inverse Model	With Inverse Model
1	0.066(0.7%)	0.026(0.29%)
5	0.091(1%)	0.030(0.33%)
10	0.14(1.56%)	0.059(0.66%)
15	0.19(2.1%)	0.082(0.91%)
20	0.22(2.4%)	0.134(1.5%)

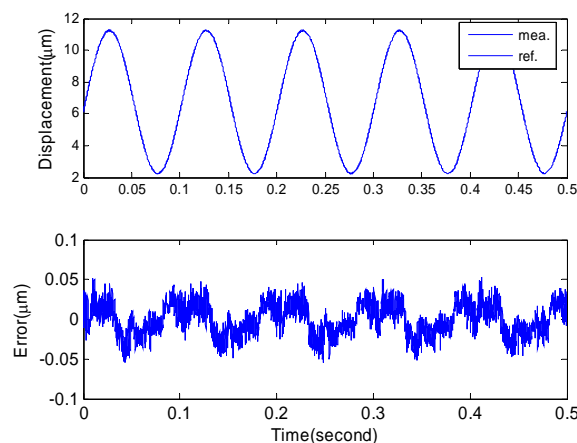


Fig. 8. Result of tracking a 10Hz sine wave.

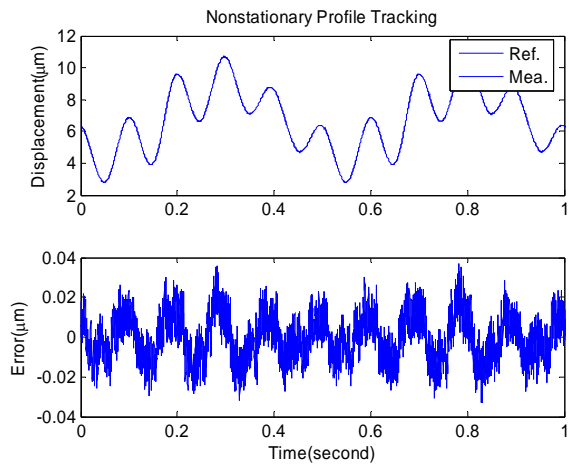


Fig. 9. Result of tracking a multi frequency, nonstationary dynamic motion profile.

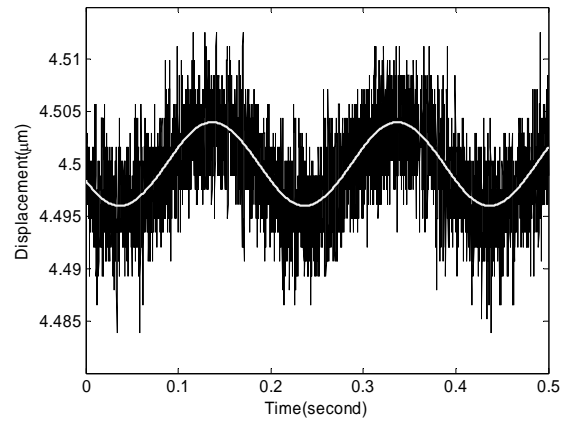


Fig. 10. Tracking resolution of the nano-stage.